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When cost metafrontiers are nonconvex in the outputs, then the production metafrontier is nonconvex: the price of a convexification strategy[☆]

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ABSTRACT

Metafrontier analysis is widely used to account for technological heterogeneity among producers. The approach involves combining a number of group-specific production possibilities sets to form a production possibilities metaset. Even though the union of the group sets normally results in a nonconvex metaset, most authors proceed as if this metaset is convex. Kerstens, O'Donnell and Van de Woestyne (2019) obtain new results on the union operator on sets under various assumptions and empirically illustrate that the popular convexification strategy is highly questionable. In this paper we transpose their results on the union operator from a production to a cost context: this is new. We then explore the extent to which convexity of the cost function is corroborated using a newly developed test. Furthermore, we check to which extent a convexification strategy is tenable when estimating a cost metafrontier. We use an original banking data set from China and the USA to illustrate the main issues. We establish that the cost function is not convex in the outputs for China and that the convexification strategy leads to potentially-biased estimates of the cost metafrontier and associated measures of efficiency.

1. Introduction

There is ample evidence that large and persistent differences in productivity levels exist across businesses (see, e.g., Syverson, 2011). For instance, in the USA a plant at the 90th percentile of the productivity distribution makes almost twice as much output with the same inputs as a plant at the 10th percentile, while in developing countries like China and India even larger productivity differences are recorded: plants at the 90th percentile of the productivity distribution make almost five times as much output given the same inputs as a plant at the 10th percentile. The question why businesses persistently differ in their measured productivity levels has attracted much attention and the causes are manifold (see the survey in Syverson (2011)). There is a wide consensus that heterogeneity in performance can be due to differences in the availability of production technologies (i.e., the techniques that are available for transforming inputs into outputs) and to differences in production environments (e.g., economic infrastructure, topography, climate, etc.). Producers often have limited direct control over these variables.

Since the publication of Nishimizu and Page (1982), there is also widespread acknowledgment that at least part of this heterogeneity in productivity may be due to technical inefficiencies (i.e., failure to make the best use of available production technologies). This has given an impetus to a large literature in economics and in operations research investigating a variety of inefficiencies using so-called frontier estimators (see, e.g., Casu et al. (2013) for an example). Within this frontier-based literature, a variety of alternative proposals have been put forward to account for heterogeneity in production frontiers. Some of the most popular methods involve the use of latent class models (e.g., Orea & Kumbhakar, 2004), the aggregation over groups or industries (e.g., Mayer & Zelenyuk, 2014), and the use of various clustering methods (e.g., Triantis et al., 2010), among others. It is our understanding that no theoretical or empirical review has carefully

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considered how these different methods account for heterogeneity in frontiers.

This contribution accounts for heterogeneity using a particular frontier estimation method that goes back to Hayami and Ruttan (1970). Those authors "call the envelope of all known and potentially discoverable activities a secular or "meta-production function"." (p. 898). Their meta-production function gives the maximum output obtainable from given inputs and a given set of production technologies (i.e., stock of knowledge). Different firms may choose a different specific technology from the set of available technologies depending on a variety of circumstances (e.g., regulation, relative prices, etc.). Parts of this literature account for the possibility of inefficiency (e.g., Lau & Yotopoulos, 1989).

These traditional ideas have been formally transposed into a production frontier framework by Battese and Rao (2002) and Battese et al. (2004). O'Donnell et al. (2008) subsequently refined some loose ends in the methodology and finalized the formal framework for making efficiency comparisons across groups of firms using both stochastic parametric and deterministic nonparametric estimation approaches. Their seminal article defines a production possibilities metaset as the union of underlying group-specific production possibilities sets (see O'Donnell et al. (2008, property R.3, p. 235)). The boundary of this metaset is referred to as a production metafrontier, and the boundaries of the group-specific sets are called group-specific production frontiers (or group frontiers).

In the last decade, metafrontier estimation has become increasingly popular and has been applied across a variety of sectors. Examples can be drawn from agriculture (e.g., Chen & Song, 2008), banking (e.g., Casu et al., 2013), hotels (e.g., Huang et al., 2013), and wastewater treatment plants (e.g., Sala-Garrido et al., 2011) to name but a few. The basic metafrontier concept has also been applied in a variety of ways: one case is the development of cost metafrontiers (e.g., Huang & Fu, 2013); another example is the computation of productivity indices relative to metafrontiers (see, e.g., Casu et al. (2013) and Huang et al. (2015) for a primal and a dual Malmquist index respectively); a final example is the development of more elaborate efficiency decompositions (see Tsekouras et al., 2017).

Group-specific production possibilities sets (PPSs) are often characterized by some standard regularity properties, one of which is convexity.[2] This convexity property is normally justified by a time divisibility argument (e.g., Shephard (1970, p. 15) or O'Donnell (2018, p. 60)). Importantly, even if group-specific PPSs are convex, the metaset defined by their union is generally nonconvex (see O'Donnell et al. (2008, p. 237)). This obvious mathematical fact is largely ignored in the productivity and efficiency literature. In the seminal article by O'Donnell et al. (2008), for example, the authors adopt a convexification strategy insofar as they estimate the production metafrontier as the boundary of a convex metaset (O'Donnell et al. (2008, p. 239), but see also, e.g., Battese and Rao (2002) and Battese et al. (2004)). A convexification strategy refers to the methodological approach of assuming convexity in a production or cost metafrontier even when the actual metafrontier may be nonconvex. Since this convexification strategy need normally not be valid, estimates of the production metafrontier risk being biased.

Kerstens et al. (2019) develop new results on the union operation on sets under various assumptions and deliver convincing empirical evidence that a convexification strategy yields statistically significant biases. In a similar vein, the empirical statistically significant biases of a convexification strategy upon the Malmquist and Hicks-Moorsteen productivity indices is documented in Jin et al. (2020).

Even though the vast majority of articles adopting a metafrontier approach appear to apply a convexification strategy, some articles do not adopt such a questionable strategy: examples include Afsharian et al. (2018), Huang et al. (2013), Sala-Garrido et al. (2011), Tiedemann et al. (2011), and (partially) Walheer (2018), among others.[3] However, these studies do not document the bias inherent in a convexification strategy.

This focus on the cost function is triggered by seminal contributions to axiomatic production theory that show that if the PPS is convex, then the cost function is convex in the outputs (see Jacobsen (1970, Proposition 5.2 (Q.9)) or Shephard (1970, Proposition 72 ($\overline{Q.11}$))). Thus, using contraposition, when the cost function is nonconvex in the outputs, then the PPS is nonconvex. Kerstens and Van de Woestyne (2021) systematically review empirical evidence and illustrate a very substantial effect of convexity on cost function estimates and on the determination of scale economies (pointing even to the possibility of contradictory results).

The purpose of this paper is to investigate for the very first time the impact of convexity on the cost function and the effect of a convexification strategy on the estimation of cost metafrontiers and associated measures of efficiency. We also provide an empirical illustration of the impact of convexity on the cost function and the effect of convexification by using Chinese and USA banking data. To the best of our knowledge, no such investigation is available in the literature while cost functions and also cost metafrontiers seem to be very widely used.[4] We anticipate two major results. First, some recent contributions have tested and rejected the convexity of the PPS for banks (e.g., Wilson (2021) and Wilson and Zhao (2023)). Based on the statistical results for cost efficiency established in Simar and Wilson (2020b), we first extend the convexity test proposed by Kneip et al. (2016) and augmented by Simar and Wilson (2020a) for PPS to the cost function: this is new. This new convexity test for the cost function is applied to Chinese and USA banking data. We find for the first time evidence that Chinese banks face a nonconvex cost function while USA banks seem to face a convex cost function: we are unaware of any other test of convexity of the cost function. Second, the cost metafrontier of Chinese and USA banking data is clearly affected by a convexification strategy yielding biased results. Thus, this almost universally adopted convexification strategy should absolutely be abandoned in the future.

Our contribution has the following structure. Section 2 develops the geometric intuition behind our claim that a convexification strategy may create biases in the estimation of cost metafrontiers. Section 3 presents a formal mathematical treatment of the metafrontier methodology with a special focus on how the union operator applies to cost functions. Section 4 explains how deterministic nonparametric frontier estimators can be used to estimate convex and nonconvex group-specific cost frontiers and associated cost metafrontiers. Section 5 presents an empirical illustration using banking data from China and the USA and makes use of advanced statistical testing tools. Finally, Section 6 summarizes our key results and wraps up with some concluding comments.

2. Production metafrontiers, cost metafrontiers and the convexification strategy: Graphical illustration

We start by reminding the reader about the intuition underlying the metafrontier approach. The metafrontier approach can be used whenever firms can be classified into groups, and when firms in different groups choose input–output combinations from potentially different PPSs. To be concrete, we consider the traditional case where all firms operating in a given period can be classified into groups according to the technologies they use. Consequently, we generally refer to group-specific sets and frontiers as technology-specific sets and frontiers.[5]

[2] A complete list with abbreviations is found in Appendix C.

[3] To put things in perspective, a Google Scholar search on 12 February 2025 obtains 12 400 results for the expression "metafrontier production".

[4] A Google Scholar search on 12 February 2025 obtains 10 900 results for the expression "metafrontier cost function".

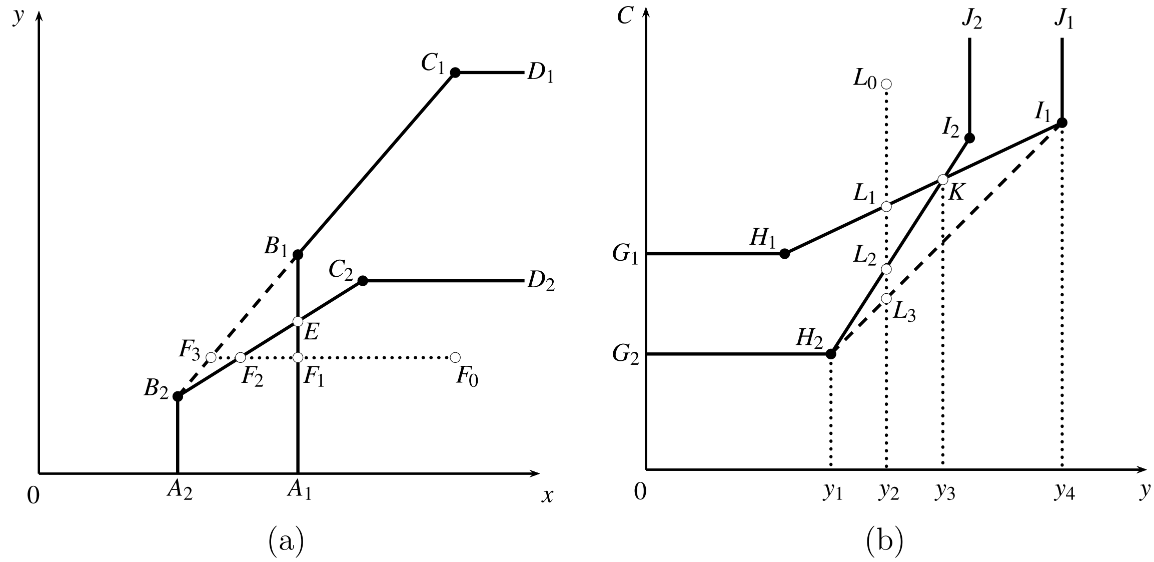

Fig. 1. (a) Production frontiers in input–output space (b) Cost frontiers in cost-output space.

We follow O'Donnell (2016, p. 328) and define a technology to be "a technique, method or system for transforming inputs into outputs ...". Fig. 1(a) illustrates technology-specific production frontiers in the simple case where firms use one input to produce one output and only two technologies are available. Let t^1 denote the set of input–output combinations that are possible using technology 1. This set is represented in Fig. 1(a) by the area above the horizontal axis and below the polyline $A_1B_1C_1D_1$; this polyline is known as the technology-1 production frontier. Similarly, let t^2 denote the set of input–output combinations that are possible using technology 2. This set is represented by the area above the horizontal axis and below the polyline $A_2B_2C_2D_2$; this polyline is known as the technology-2 production frontier. Note that both of the technology-specific sets t^1 and t^2 are convex. The convexity assumption on these sets implies perfect time-divisibility, i.e., that it is meaningful to combine observations belonging to each set in a linear way.[6]

Fig. 1(a) also illustrates the concept of a production metafrontier. Let T denote the set of input–output combinations that are possible using the two technologies that are available. This metaset is given by the union of t^1 and t^2. It is represented in Fig. 1(a) by the area above the horizontal axis and below the polyline $A_2B_2EB_1C_1D_1$. This polyline is known as the period-t production metafrontier. Clearly, this metaset T is nonconvex. The convexification strategy involves convexifying T by adding the points in the triangle $B_2EB_1B_2$: these points are only feasible (i.e., convexification is only valid) if it is possible to use technology 1 some of the time and technology 2 the rest of the time.

Assuming perfect time divisibility, it may make sense to compare any two points within the set t^1 to learn how one can improve the performance of firms that use technology 1, and it may make sense to do the same for any two points within the set t^2. But, if firms are locked into using either technology 1 or technology 2, then one cannot learn how to improve their performance by a comparison of two points belonging to different sets.

In other words, it should be realized that a convexification strategy is to some extent self-contradictory: it runs counter the very idea of distinguishing between different production possibilities sets and only allowing for the convexity of each set. Otherwise stated, the union operator on technology-specific sets does not preserve convexity of the resulting metaset.

We can now consider the measurement of efficiency with respect to these frontiers. Consider the firm operating at point F_0, and recall that the firm can only use one technology (not a combination of both). Recall that in this case the production metafrontier is given by the polyline $A_2B_2EB_1C_1D_1$. Assume that the firm uses technology 1, then it can minimize the input required to produce its output by moving to point F_1 on the technology-1 production frontier. If instead it had used technology 2, then it could have minimized input use by moving to point F_2 on the metafrontier. Full technical efficiency would have required that it had opted to use technology 2 and operate at F_2. The input-oriented technical efficiency (ITE) of a firm is an input-oriented measure of the distance from an observed point to a point on the metafrontier. The ITE of the firm operating at F_0, for example, is computed as the input used at F_2 divided by the input used at F_0. The ITE of a firm can be broken into the product of an input-oriented metatechnology ratio (IMR) and a measure of residual input-oriented technical efficiency (RITE). The IMR is an input-oriented measure of how well the firm has initially chosen its technology among the available options, while RITE is an input-oriented measure of how well its chosen technology has been used. Assuming that the firm operating at F_0 had chosen technology 1, for example, then its IMR would be computed as the input used at F_2 divided by the input used at F_1; its RITE would be computed as the input used at F_1 divided by the input used at F_0.

It is important to note that if the firm counterfactually had been able to use technology 1 some of the time and technology 2 the rest of the time, then the production metafrontier would have been given by the polyline $A_2B_2B_1C_1D_1$. In this counterfactual case, the firm would have been able to minimize input use by moving to F_3 on the metafrontier. The ITE of the firm operating at F_0 would then have been computed as the input used at F_3 divided by the input used at F_0. Assuming that the firm had chosen technology 1, then the IMR would have been computed as the input used at F_3 divided by the input used at F_1. Its RITE would still have been computed as the input used at F_1 divided by the input used at F_0. Hence, it follows that incorrectly convexifying the metaset leads to downwardly biased measures of ITE and IMR (but not RITE).

5 Translating our results to situations where firms are classified according to other criteria is straightforward but may involve slightly different terminology. For instance, if firms are classified into groups according to the production environments in which they operate, then group-specific sets and frontiers might be referred to as environment-specific sets and frontiers; if production environments are viewed as states of nature, then they might be referred to as state-contingent sets and frontiers.

6 Shephard (1970, p. 15) states that convexity is only "valid for time divisibly-operable technologies".

In this contribution we consider the metafrontier approach in a cost frontier context. Fig. 1(b) illustrates technology-specific cost frontiers in the simple case where price-taking firms produce one output and only two technologies are available. Let $c^1(y,w)$ denote the minimum cost of producing y when using technology 1 and when input prices are given by w. This function is represented in Fig. 1(b) by the polyline $G_1H_1I_1J_1$. This polyline is known as the technology-1 cost frontier: all cost-output combinations to the right of the vertical axis and above this frontier are possible when using technology 1. Similarly, let $c^2(y,w)$ denote the minimum cost of producing y when using technology 2 and when input prices are given by w. This function is represented in Fig. 1(b) by the polyline $G_2H_2I_2J_2$. This polyline is known as the technology-2 cost frontier: all cost-output combinations to the right of the vertical axis and above this frontier are possible when using technology 2.

Fig. 1(b) also illustrates the concept of a cost metafrontier. Let $C(y,w)$ denote the minimum cost of producing y when using the two technologies available and when input prices are given by w. This function is represented in Fig. 1(b) by the polyline $G_2H_2KI_1J_1$. This polyline is known as the cost metafrontier: all cost-output combinations to the right of the vertical axis and above this metafrontier are possible when using the available technologies. Clearly, this set of cost-output combinations is nonconvex. The convexification strategy now involves adding the points in the triangle $H_2I_1KH_2$. Again, points in this triangle are only feasible (i.e., the convexification strategy is only valid) if it would be possible to use technology 1 some of the time and technology 2 the rest of the time.

But, as argued for the metaset above, this convexification strategy is self-contradictory and does not allow to learn from proper comparisons among the group cost functions.

We can now consider the measurement of cost efficiency with respect to these cost frontiers. Consider the firm operating at point L_0, and again recall that this firm is only able to use one technology (not a combination of both). Recall that in this case the cost metafrontier is given by the polyline $G_2H_2KI_1J_1$. Assume that the firm uses technology 1, then it minimizes the cost of producing its output by moving to point L_1 on the technology-1 cost frontier. If instead it had used technology 2, then it could have minimized cost by moving to point L_2 on the metafrontier. Full cost efficiency would have required that it use technology 2 and operate at L_2. The cost efficiency (CE) of a firm is a cost-oriented measure of the distance from an observed point to a point on the metafrontier. The CE of the firm operating at L_0, for example, is computed as the cost at L_2 divided by the cost at L_0. The CE of a firm can be broken into the product of a cost-oriented metatechnology ratio (CMR) and a measure of residual cost efficiency (RCE). The CMR is a cost-oriented measure of how well the firm has chosen its technology, while RCE is a cost-oriented measure of how well its chosen technology has been used. Assuming the firm operating at L_0 has chosen technology 1, for example, then its CMR is computed as the cost at L_2 divided by the cost at L_1: its RCE is computed as the cost at L_1 divided by the cost at L_0.

Again, it is important to consider the counterfactual case where the firm had been able to use technology 1 some of the time and technology 2 the rest of the time. In this case, the cost metafrontier would have been given by the polyline $G_2H_2I_1J_1$. Consequently, the firm would have been able to minimize cost by moving to point L_3. The CE of the firm operating at L_0 would then have been computed as the cost at L_3 divided by the cost at L_0. If the firm had chosen technology 1, then the CMR would have been computed as the cost at L_3 divided by the cost at L_1. Its RCE would still have been computed as the cost at L_1 divided by the cost at L_0. It follows that incorrectly convexifying the cost-output metaset leads to downwardly biased measures of CE and the CMR (but not RCE).

This discussion of convexification can be linked to a specific property of the cost function that is worth spelling out: if PPSs are convex, then cost functions are convex in outputs (see, e.g., some seminal contributions to axiomatic production theory like Jacobsen (1970, Proposition 5.2) or Shephard (1970, p. 227)). This empirical property of the cost function seems rarely tested. This general property has been sharpened by Briec et al. (2004) who establish that (i) cost functions estimated on convex PPSs yield lower or equal cost estimates compared to cost functions estimated on nonconvex PPSs; and (ii) both types of cost functions coincide for the single output and constant returns to scale case. Obviously, similar properties exist for the revenue function and the profit function: except for the long-run profit function, all other dual specifications are affected by the convexity or not of the PPS.

If the production metafrontier is nonconvex, which is normally the case, then the cost metafrontier is generally nonconvex in outputs (see Fig. 1(b)). To the best of our knowledge, there are only two articles that estimate a cost metafrontier (or similar dual function) without using a convexification strategy: Campos-Alba et al. (2020) and Pérez-López et al. (2016) estimate a cost metafrontier under the assumption that neither the group cost frontiers nor the cost metafrontier are convex.[7] Instead, most researchers adopt a convexification strategy. For example, Bos and Schmiedel (2007) estimate cost metafrontiers using parametric estimators and a convexification strategy, while Huang et al. (2015) estimate cost metafrontiers (or similar dual functions) using nonparametric estimators and a convexification strategy. None of these authors test the validity of their convexification strategy. A benign interpretation of this state of affairs is that most authors seem to believe that using a convexification strategy when estimating a cost metafrontier is innocuous and does not lead to any bias. This explains the title of our contribution: we are interested in whether the convexification strategy leads to biased estimates of the true nonconvex cost metafrontier and associated measures of efficiency.

3. Production metafrontiers and cost metafrontiers: Mathematical analysis

We begin our formal mathematical treatment of the metafrontier methodology by introducing some useful basic mathematical notions and notation.

3.1. Mathematical preliminaries

Let $m_A \in \bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ denote the infimum of a set $A \subseteq \mathbb{R}$. Note that $m_A = -\infty$ if the set A is unbounded to the left, and $m_A = +\infty$ if A is empty or unbounded to the right. We then have the following proposition.

Proposition 3.1. *Consider a real-valued function $f : \mathbb{R}^n \to \mathbb{R}$ with $n \in \mathbb{N}$ variables. Let A and B be subsets of $\mathrm{dom}(f)$. Let m_A, m_B and m_{AB} denote the infimum of $\{f(x) \mid x \in A\}$, $\{f(x) \mid x \in B\}$ and $\{f(x) \mid x \in A \cup B\}$, respectively. Then, the following holds true:*

(a) If $A \subseteq B$, then $m_A \geq m_B$;
(b) $m_{AB} = \min\{m_A, m_B\}$.

This proposition is common knowledge: thus, we omit its proof. Proposition 3.1 states that if set A is a subset of set B, then the infimum of a real-valued function over set B is smaller than the infimum of that same real-valued function over set A. The infimum of a real-valued function over the union of two sets is the minimum of the respective infima of this real-valued function over both sets separately.

[7] Obviously, as mentioned above, also some contributions using a long-run profit function approach when estimating a metafrontier do not introduce any bias, since the effect of nonconvexity of the PPS is logically indistinguishable.

3.2. Technologies and cost frontiers

Technologies can be represented by technology-specific PPSs. The gth technology-specific production possibilities set (TPPS), for example, is the set containing all input–output combinations that are possible using technology g. Denote the number of inputs and outputs by $M \in \mathbb{N}$ and $N \in \mathbb{N}$, respectively, and denote by $x \in \mathbb{R}_+^M$ and $y \in \mathbb{R}_+^N$ the vectors of inputs and outputs, respectively. Mathematically, the gth TPPS is:

$$t^g = \{(x, y) \in \mathbb{R}_+^M \times \mathbb{R}_+^N \mid x \text{ with technology } g \text{ can produce } y\}. \tag{1}$$

Loosely speaking, the boundary of this set is the gth technology-specific production frontier. In the literature, it is also common to refer to this boundary as the gth group frontier.

It is common to make one or more of the following assumptions regarding the TPPS defined by (1):

(T.1) $(x, 0) \in t^g$ for all $x \in \mathbb{R}_+^M$.
(T.2) If $(0, y) \in t^g$, then $y = 0$.
(T.3) t^g is a closed subset of $\mathbb{R}_+^M \times \mathbb{R}_+^N$.
(T.4) If $(x, y) \in t^g$ and $(x', -y') \geq (x, -y)$, then $(x', y') \in t^g$.
(T.5) t^g is a convex set.
(T.6) If $(x, y) \in t^g$, then $\delta(x, y) \in t^g$ for all $\delta \geq 0$.

In words, these assumptions state that: (i) inactivity is possible, (ii) there is no free lunch, (iii) the set of feasible input–output combinations contains all the points on its boundary (closedness), (iv) inputs and outputs are freely (or strongly) disposable, (v) the TPPS is convex, and (vi) the technology-specific production frontier exhibits constant returns to scale. For more details on these assumptions see, for example, O'Donnell (2018, p. 55–63).

Under weak regularity conditions, technologies can be represented by technology-specific cost functions. The gth technology-specific cost function (TCF), for example, gives the minimum cost of producing a given output vector when using technology g and facing given input prices. Mathematically, this leads to the following definition:

Definition 3.1. The TCF that gives the minimum cost of producing $y \in \mathbb{R}_+^N$ when using technology g and facing prices $w \in \mathbb{R}_+^M$ is defined as $c^g : \mathbb{R}_+^N \times \mathbb{R}_+^M \to \mathbb{R}_+ : (y, w) \mapsto c^g(y, w) = \inf\{w'x \mid (x, y) \in t^g\}$.

With this definition we can now state the following proposition.

Proposition 3.2. *Consider two technologies g and h, a given output level $y \in \mathbb{R}_+^N$ and a vector of input prices $w \in \mathbb{R}_+^M$. If $t^g \subseteq t^h$, then $c^g(y, w) \geq c^h(y, w)$.*

The proof of this proposition and all other propositions is found in Appendix A. Thus, the cost function representing a smaller PPS always takes a value that is greater than or equal to the value taken by a cost function representing a larger PPS.

3.3. Metatechnologies and cost metafrontiers

The set of technologies that exist in a given period is sometimes referred to as a technology set (e.g. O'Donnell, 2018, p. 87). In the metafrontier literature, technology sets are more often referred to as metatechnologies. Metatechnologies can be represented by metatechnology-specific production possibilities sets. The metatechnology-specific production possibilities set (MPPS), for example, is the set containing all input–output combinations that are possible using the metatechnology (i.e., using the technologies that exist in a given period).

To formalize these ideas, let $\Gamma = \{1, \ldots, P\}$ denote the set of $P \in \mathbb{N}$ technologies that exist in a given period. Then Γ can be represented by $T = \bigcup_{g=1}^{P} t^g = \bigcup_{g \in \Gamma} t^g$. Using the definition of the union operator, note that this MPPS can also be defined as

$$T = \{(x, y) \in \mathbb{R}_+^M \times \mathbb{R}_+^N \mid \exists g \in \Gamma : x \text{ and technology } g \text{ can produce } y\}. \tag{2}$$

This metaset inherits many of its properties from the properties of the TPPSs. This is explicit in the following proposition.

Proposition 3.3. *Consider the metatechnology Γ. If t^g satisfies assumption (T.n), with $n \in \{1, 2, 3, 4, 6\}$, for all $g \in \Gamma$, then T also satisfies (T.n).*

Remark 3.1. If t^g satisfies convexity (i.e., (T.5)) for all $g \in \Gamma$, then T does not necessarily satisfy assumption (T.5). This statement is easily illustrated in Fig. 1(a).

It is common to represent metatechnologies using metatechnology-specific cost functions. The metatechnology-specific cost function (MCF), for example, gives the minimum cost of producing a given output vector when using the metatechnology and facing given input prices. Mathematically, this leads to the following definition:

Definition 3.2. The MCF that gives the minimum cost of producing $y \in \mathbb{R}_+^N$ when using the metatechnology and facing input prices $w \in \mathbb{R}_+^M$ is defined as $C : \mathbb{R}_+^N \times \mathbb{R}_+^M \to \mathbb{R}_+ : (y, w) \mapsto C(y, w) = \inf\{w'x \mid (x, y) \in T\}$.

While Definition 3.2 assumes fixed input prices w for theoretical derivation, empirical estimation incorporates heterogeneous input prices to better capture the realities of firm-level cost variations. We therefore have the following remark for the empirical applications.

Remark 3.2. If firm i faces heterogeneous input prices $w_i \in \mathbb{R}_+^M$, the MCF that gives the minimum cost of producing $y_i \in \mathbb{R}_+^N$ when using the metatechnology is defined as $C : \mathbb{R}_+^N \times \mathbb{R}_+^M \to \mathbb{R}_+ : (y_i, w_i) \mapsto C(y_i, w_i) = \inf\{w_i'x \mid (x, y_i) \in T\}$.

The MCF satisfies the following relations.

Proposition 3.4. *Consider the metatechnology Γ, a given output level $y \in \mathbb{R}_+^N$ and a vector of input prices $w \in \mathbb{R}_+^M$.*

(a) $\forall g \in \Gamma : c^g(y, w) \geq C(y, w)$;
(b) $C(y, w) = \min\{c^g(y, w) \mid g \in \Gamma\}$.

Part (a) of this proposition says that the value of every TCF is larger than or equal to the value of the MCF: this is because every TPPS is contained in the MPPS. Part (b) of this proposition says that in order to determine the value of the MCF it suffices to get the minimum of all available TCFs.

Note that, similar to distance functions (see Kerstens et al., 2019), TCFs and MCFs are not always well-defined (in the sense of resulting in a finite value). To illustrate this, consider the firm operating at point I_1 in Fig. 1(b). It is not possible to produce output y_4 using technology 2, so $c^2(y_4, w) = +\infty$. Note that even though the technology-2 cost function is not well-defined, the MCF is still well-defined: as a result of Proposition 3.4(b), $C(y_4, w) = \min\{c^1(y_4, w), +\infty\} = c^1(y_4, w)$.

Proposition 3.4(b) also implies that the MCF need not be convex in outputs. Again, Fig. 1(b) can be used to illustrate. Consider the output level y_3 of observation K. Since this output level is located between the output levels y_1 and y_4 of observations H_2 and I_1, respectively, there exists some $\alpha \in (0, 1)$ such that $y_3 = \alpha y_1 + (1 - \alpha) y_4$. According to Proposition 3.4(b), $C(y_3, w) = \min\{c^1(y_3, w), c^2(y_3, w)\} = c^1(y_3, w) = c^2(y_3, w)$. It is easy to show that this value is larger than the value $\alpha C(y_1, w) + (1 - \alpha) C(y_4, w) = \alpha c^2(y_1, w) + (1 - \alpha) c^1(y_4, w)$. By definition, this illustrates the nonconvexity of the MCF in this example.

Finally, recall from Section 2 that the CE of a firm can be decomposed into the product of a CMR and a measure of RCE. Mathematically, the CE of a firm that faces input prices w and uses inputs x to produce outputs y is $\mathrm{CE}(x, y, w) = C(y, w)/w'x$. If a firm uses technology g, then its CMR and RCE are $\mathrm{CMR}^g(y, w) = C(y, w)/c^g(y, w)$ and $\mathrm{RCE}^g(x, y, w) =$

$c^g(y,w)/w'x$. Since $w'x \geq c^g(y,w) \geq C(y,w)$, it is obvious that all three measures lie in the closed unit interval. Moreover, we have the following decomposition:

$$\mathrm{CE}(x,y,w) = \mathrm{CMR}^g(y,w) \cdot \mathrm{RCE}^g(x,y,w). \tag{3}$$

This implies $\mathrm{RCE}^g(x,y,w) = \mathrm{CE}(x,y,w)/\mathrm{CMR}^g(y,w)$. Thus, RCE can be viewed as the component of cost efficiency that remains after accounting for the CMR (hence the term 'residual'). Related cost-oriented measures of performance are found elsewhere in the literature: e.g., our CMR is the reciprocal of the 'cost gap ratio' defined by Huang et al. (2015, p. 325). Those same authors also exploit the traditional distinction between technical and allocative efficiencies to decompose their equivalent of RCE. Note furthermore that the convexification strategy consisting in convexifying the metatechnology when computing the cost function affects the whole decomposition (3), except for the component $\mathrm{RCE}^g(x,y,w)$.

4. Nonparametric frontier estimators

In the next two subsections, we examine the consequences of convexification for well-known nonparametric estimators of nonconvex and convex PPSs and associated cost functions. Suppose we have $n \in \mathbb{N}$ observed input–output combinations with which to estimate the MPPS. We introduce the following notation. The observed input–output combinations used to estimate the MPPS are denoted $(x_1,y_1), \ldots, (x_n,y_n) \in \mathbb{R}^M_+ \times \mathbb{R}^N_+$. The nonparametric estimator of the gth TPPS only uses $n^g \leq n$ of these observations. To identify these particular observations, consider the one-to-one index function $\phi_g : \{1,\ldots,n^g\} \to \{1,\ldots,n\}$. Then, $(x_{\phi_g(i)}, y_{\phi_g(i)})$ denotes the ith observation in the set of observations used to estimate the gth TPPS. For example, consider the case where the nonparametric estimator of the gth TPPS only uses the four observations (x_2,y_2), (x_4,y_4), (x_5,y_5) and (x_7,y_7). Then, $n^g = 4$ and $\phi_g : \{1,2,3,4\} \to \{1,\ldots,n\}$ with $\phi_g(1) = 2$, $\phi_g(2) = 4$, $\phi_g(3) = 5$ and $\phi_g(4) = 7$.

4.1. Nonconvex PPSs and related cost functions

We begin by considering the estimation of nonconvex PPSs under the assumption of either variable or constant returns to scale. First, if all TPPSs are nonconvex (NC) and their corresponding frontiers exhibit variable returns to scale (VRS), then an asymptotically unbiased estimator of the gth TPPS is:

$$t^g_{NC,VRS} = \Big\{(x,y) \in \mathbb{R}^M_+ \times \mathbb{R}^N_+ \mid \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0,1\}\Big\}. \tag{4}$$

Importantly, the constraints $\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1$ and $\lambda_{\phi_g(i)} \in \{0,1\}$ ensure that only one activity vector $\lambda_{\phi_g(i)}$ is nonzero (and equal to one); except in restrictive special cases, this means that the estimated TPPS is nonconvex. The estimator defined by (4) is commonly known as a free disposal hull (FDH) estimator. The associated FDH estimator of the MPPS is: $T_{NC,VRS} = \bigcup_{g \in \Gamma} t^g_{NC,VRS}$.[8]

Second, if all TPPSs are NC and their corresponding frontiers exhibit constant returns to scale (CRS), then an asymptotically unbiased estimator of the gth TPPS is:

$$t^g_{NC,CRS} = \Big\{(x,y) \in \mathbb{R}^M_+ \times \mathbb{R}^N_+ \mid \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0,1\}, \delta \geq 0\Big\}. \tag{5}$$

[8] Campos-Alba et al. (2020) and Pérez-López et al. (2016) consider a robust version of this estimator that allows for outliers.

Observe that this estimator includes a scaling parameter δ; this parameter allows for an unlimited scaling of all n^g observations determining the TPPS so as to embody the assumption of CRS. The associated estimator of the MPPS is $T_{NC,CRS} = \bigcup_{g \in \Gamma} t^g_{NC,CRS}$.

Definitions 3.1 and 3.2 can now be used to motivate FDH estimators of TCFs and MCFs. An overview is given in the following definition.

Definition 4.1. For the metatechnology Γ, some output level $y \in \mathbb{R}^N_+$ and a given vector of input prices $w \in \mathbb{R}^M_+$, the following estimators can be introduced:

(a) If all TPPSs are NC and their corresponding frontiers exhibit VRS, then an asymptotically unbiased estimator of:

(i) the gth TCF is $c^g_{NC,VRS}(y,w) = \inf\{w'x \mid (x,y) \in t^g_{NC,VRS}\}$;
(ii) the MCF is $C_{NC,VRS}(y,w) = \inf\{w'x \mid (x,y) \in T_{NC,VRS}\}$.

(b) If all TPPSs are NC and their corresponding frontiers exhibit CRS, then an asymptotically unbiased estimator of:

(i) the gth TCF is $c^g_{NC,CRS}(y,w) = \inf\{w'x \mid (x,y) \in t^g_{NC,CRS}\}$;
(ii) the MCF is $C_{NC,CRS}(y,w) = \inf\{w'x \mid (x,y) \in T_{NC,CRS}\}$.

Relations between several of these estimators are summarized in the following proposition.

Proposition 4.1. *Consider the metatechnology Γ, some output level $y \in \mathbb{R}^N_+$ and a given vector of input prices $w \in \mathbb{R}^M_+$. Then the following relations hold true for all $g \in \Gamma$:*

(a) $c^g_{NC,VRS}(y,w) \geq C_{NC,VRS}(y,w)$, $\quad c^g_{NC,CRS}(y,w) \geq C_{NC,CRS}(y,w)$;

(b) $c^g_{NC,VRS}(y,w) \geq c^g_{NC,CRS}(y,w)$, $\quad C_{NC,VRS}(y,w) \geq C_{NC,CRS}(y,w)$;

(c) $C_{NC,VRS}(y,w) = \min\{c^g_{NC,VRS}(y,w) \mid g \in \Gamma\}$;

(d) $C_{NC,CRS}(y,w) = \min\{c^g_{NC,CRS}(y,w) \mid g \in \Gamma\}$;

(e) $$c^g_{NC,VRS}(y,w) = \min\Big\{ w'x \mid \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0,1\}\Big\};$$

(f) $$c^g_{NC,CRS}(y,w) = \min\Big\{ w'x \mid \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0,1\}, \delta \geq 0\Big\};$$

(g) $$C_{NC,VRS}(y,w) = \min\Big\{ w'x \mid \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0,1\}\Big\};$$

(h) $$C_{NC,CRS}(y,w) = \min\Big\{ w'x \mid \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0,1\}, \delta \geq 0\Big\}.$$

Part (a) of this proposition says that estimated TCFs always take values that are larger than or equal to the values taken by estimated MCFs, while part (b) simply reminds us that cost functions estimated under a VRS assumption always take values that are larger than or equal to the values taken by cost functions estimated under a CRS assumption. Parts (c) and (d) say that, irrespective of the returns to scale assumption, the estimated values of MCFs can be computed as the minima of the estimated values of the relevant TCFs. Parts (e) and (f) define the estimated TCFs under the assumptions of VRS and CRS. Finally, parts (g) and (h) reveal that MCF is estimated using all observations associated with all the technologies available in a given period.

Finally, Proposition 4.1 reveals that estimating cost functions associated with nonconvex PPSs involves solving various linear and nonlinear binary mixed integer programs. For details of alternative solution strategies,including the fastest implicit enumeration algorithms over all observations n: see Kerstens and Van de Woestyne (2014).

4.2. Convex PPSs and related cost functions

We now consider the estimation of convex PPSs under the assumption of either VRS or CRS. First, if all TPPSs are convex and their corresponding frontiers exhibit VRS, then an asymptotically unbiased estimator of the gth TPPS is:

$$t^g_{C,VRS} = \Big\{ (x,y) \in \mathbb{R}^M_+ \times \mathbb{R}^N_+ \mid \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \geq 0 \Big\}. \tag{6}$$

This estimator is the convexified version of (4). It differs from (4) in that the nonnegative activity (or intensity) variables ($\lambda_{\phi_g(i)}$) are no longer restricted to be binary integers. The estimator defined by (6) is commonly known as a data envelopment analysis (DEA) estimator. The associated convex estimator of the MPPS is: $T_{C,VRS} = \bigcup_{g \in \Gamma} t^g_{C,VRS}$.

Second, if all TPPSs are convex and their corresponding frontiers exhibit CRS, then an asymptotically unbiased estimator of the gth TPPS is:

$$t^g_{C,CRS} = \Big\{ (x,y) \in \mathbb{R}^M_+ \times \mathbb{R}^N_+ \mid \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \lambda_{\phi_g(i)} \geq 0 \Big\}. \tag{7}$$

This estimator is the convexified version of (5). Again, it differs from (5) in that the nonnegative activity variables are no longer restricted to be binary integers. Note that an additional transformation (i.e., rewrite $\delta\lambda_{\phi_g(i)}$ in (5) as $\lambda'_{\phi_g(i)}$ and rename as $\lambda_{\phi_g(i)}$) is needed to obtain the exact result in (7). The associated convex estimator of the MPPS is $T_{C,CRS} = \bigcup_{g \in \Gamma} t^g_{C,CRS}$.

Definitions 3.1 and 3.2 can now be used to motivate convex estimators of TCFs and MCFs. An overview is given in the following definition.

Definition 4.2. For the metatechnology Γ, some output level $y \in \mathbb{R}^N_+$ and a given vector of input prices $w \in \mathbb{R}^M_+$, the following estimators can be introduced:

(a) If all TPPSs are convex and their corresponding frontiers exhibit VRS, then an asymptotically unbiased estimator of:

- (i) the gth TCF is $c^g_{C,VRS}(y,w) = \inf\{w'x \mid (x,y) \in t^g_{C,VRS}\}$;
- (ii) the MCF is $C_{C,VRS}(y,w) = \inf\{w'x \mid (x,y) \in T_{C,VRS}\}$.

(b) If all TPPSs are convex and their corresponding frontiers exhibit CRS, then an asymptotically unbiased estimator of:

- (i) the gth TCF is $c^g_{C,CRS}(y,w) = \inf\{w'x \mid (x,y) \in t^g_{C,CRS}\}$;
- (ii) the MCF is $C_{C,CRS}(y,w) = \inf\{w'x \mid (x,y) \in T_{C,CRS}\}$.

Relations between several of these estimators are summarized in the following proposition.

Proposition 4.2. *Consider the metatechnology Γ, some output level $y \in \mathbb{R}^N_+$ and a given vector of input prices $w \in \mathbb{R}^M_+$. Then the following relations hold true for all $g \in \Gamma$:*

(a) $c^g_{C,VRS}(y,w) \geq C_{C,VRS}(y,w)$, $\quad c^g_{C,CRS}(y,w) \geq C_{C,CRS}(y,w)$;

(b) $c^g_{C,VRS}(y,w) \geq c^g_{C,CRS}(y,w)$, $\quad C_{C,VRS}(y,w) \geq C_{C,CRS}(y,w)$;

(c) $C_{C,VRS}(y,w) = \min\{c^g_{C,VRS}(y,w) \mid g \in \Gamma\}$;

(d) $C_{C,CRS}(y,w) = \min\{c^g_{C,CRS}(y,w) \mid g \in \Gamma\}$;

(e) $c^g_{C,VRS}(y,w) = \min \Big\{ w'x \mid \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \geq 0 \Big\}$;

(f) $c^g_{C,CRS}(y,w) = \min \Big\{ w'x \mid \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \lambda_{\phi_g(i)} \geq 0 \Big\}$;

(g) $C_{C,VRS}(y,w) \geq \min \Big\{ w'x \mid \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \geq 0 \Big\}$;

(h) $C_{C,CRS}(y,w) \geq \min \Big\{ w'x \mid \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \lambda_{\phi_g(i)} \geq 0 \Big\}$.

(i) If $N = 1$, (i.e., only one output is produced), then $C_{C,CRS}(y,w) = \min \Big\{ w'x \mid \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{g \in \Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \lambda_{\phi_g(i)} \geq 0 \Big\}$.

Note that Propositions 4.1 and 4.2 are rather similar. However, important differences can be observed when comparing parts (g) and (h). The equality observed in the nonconvex case is now replaced by an inequality in the convex case. These results are similar to the ones in Proposition 5.5 in Kerstens et al. (2019). We stress that equality is not true in general unless CRS is assumed and only one output is produced (i.e., result (i) of Proposition 4.2). Also note that the cost functions on the right-hand sides of (g) and (h) of Proposition 4.2 correspond with those of the convexified MPPSs. Hence, costs computed by incorrectly convexifying a MPPS systematically underestimate true costs.

Finally, Proposition 4.2 reveals that estimating cost functions associated with convex production possibilities sets involves solving linear programming problems for each evaluated observation, as discussed in the mainstream efficiency literature (e.g., see O'Donnell (2018, p. 226)). This implies that metatechnology specific cost functions in the convex case can only be computed via a sequence of linear programs using Proposition 3.4(b), while the nonconvex counterparts only necessitate computing simple implicit enumeration algorithms over all observations n. It remains an open question whether in the convex case a single linear program can be found to do the job.

5. Empirical illustration

In this section we use original banking data from China (CN) and the United States (USA) to illustrate the potential effects of a convexification strategy on estimates of cost metafrontiers and associated measures of efficiency.

5.1. Data set: Banks from China and the United States

We retrieve the banking data for CN and the USA in 2019 from the BankScope database. Our sample comprises the largest banks encompassing 124 banks from CN and 153 banks from the USA. Following the intermediation approach, we assume that banks use three inputs to produce three outputs. The three inputs are deposits (x_1), labor (x_2), and physical capital (x_3). Their respective input prices are determined by the ratio of input expenses to the corresponding inputs, denoted as w_1, w_2, and w_3, respectively.[9] The three outputs are loans (y_1),

[9] Since the number of employees is unavailable for many banks in our sample, we adopt the approach of Beccalli et al. (2015), where the ratio of labor expenses to total assets is used as a proxy for labor price and total assets as a proxy for labor.

Table 1
Summary statistics for the CN and USA banks.

Variable	Minimum	Q1	Median	Mean	Q3	Maximum
124 CN banks						
y_1	42.937	9 289.814	15 750.933	124 116.189	36 659.041	2 405 188.184
y_2	48.148	3 090.382	6 626.673	50 094.086	16 053.784	1 057 053.375
y_3	0.000	1 596.715	4 251.240	32 476.381	10 643.788	441 649.774
x_1	92.804	15 013.189	29 290.715	196 198.100	72 066.293	3 701 724.024
x_2	330.115	17 606.624	33 176.844	228 003.751	80 235.759	4 309 351.040
x_3	0.514	69.544	168.420	1 665.048	497.133	41 013.453
w_1	0.010	0.022	0.026	0.026	0.029	0.042
w_2	0.002	0.004	0.005	0.005	0.006	0.020
w_3	0.205	0.456	0.634	1.804	1.097	18.556
153 USA banks						
y_1	1 126.664	4 458.623	9 671.747	47 214.852	23 271.356	969 829.000
y_2	150.996	1 180.880	2 785.468	27 034.831	7 563.811	1 084 769.000
y_3	18.614	1 827.726	3 755.381	45 562.285	11 889.265	1 174 417.000
x_1	2 839.178	5 240.233	11 574.706	69 351.504	28 975.669	1 776 586.000
x_2	3 614.957	6 173.877	13 495.944	85 878.343	34 105.305	2 337 646.000
x_3	0.007	67.134	149.366	813.153	340.515	22 432.000
w_1	0.000	0.007	0.010	0.011	0.013	0.036
w_2	0.000	0.010	0.012	0.013	0.014	0.050
w_3	0.214	0.599	0.815	13 919.060	1.299	2 128 807.000

securities (y_2), and off-balance items (y_3). Table 1 summarizes the descriptive statistics for the inputs, the outputs and the input prices for both CN and USA banks. All monetary values are reported in constant million USA dollars.

Table 1 indicates that CN banks tend to be larger than USA banks, with both median and mean values of loans and deposits surpassing those of USA banks. Nonetheless, USA banks typically have a greater amount of off-balance sheet items. As a result, there is significant heterogeneity between CN and USA banks. In this illustration, it is rational to assume that the banks in CN and USA have different technologies. The differences between CN and USA banks can be multifaceted and can include variations in market structures, management practices, technological advancement, economic conditions, and regulatory frameworks. By definition, our metatechnology is $\Gamma = \{1, 2\}$.

The traditional economic understanding of the bank's production makes us believe that it *may* be possible for the manager of a given bank to use a given input vector to produce a given level of output for some time within the production period, and then use a different input vector to produce a different level of output for the rest of the time. This suggests that each TPPS *may* be convex. Consequently, we begin by estimating convex TPPSs and associated TCFs using a nonparametric convex estimator. Given the different types of regulatory frameworks and market structures involved in the production, it is also our understanding that the manager of a given bank cannot normally generate the outputs by using convex combinations of technologies in CN and USA. This suggests that the MPPS should not be convexified. It is now an open question to check how a convexification strategy of the MPPS approximates the true nonconvex MPPS. We are particularly interested in the effects of the convexification strategy on estimates of efficiency.

It is widely assumed in the banking literature that the bank's production process in general is characterized by convex PPSs, and that the boundaries of those sets are linear. However, recently several contributions have tested and rejected the convexity of the PPS for banks (see, e.g., Wilson and Zhao (2023) for CN banks and Wilson (2021) for USA banks). If PPSs are nonconvex, then there is no reason to suppose that cost functions are convex in the outputs. Economists who take this seriously would presumably want to estimate TPPSs that are not convex. To satisfy the curiosity of these economists, we also estimate nonconvex TPPSs and associated TCFs using a nonparametric FDH estimator.

5.2. Empirical results

Descriptive statistics for the estimates of $\mathrm{CE}(x, y, w)$, $\mathrm{RCE}^g(x, y, w)$ and $\mathrm{CMR}^g(y, w)$ are reported in the columns labeled C-NC and NC-NC in Table 2. The acronym C-NC indicates that the TPPSs are convex but the MPPS is not, while the acronym NC-NC indicates that neither the TPPSs nor the MPPS are convex. All estimates have been obtained under the assumptions that production frontiers exhibit VRS. In Table 2, both C-NC and NC-NC results are reported in two blocks of four columns each, where the last column contains the number of infeasible solutions.[10] Turning to the explanation of the rows in Table 2, the first block of results contains summary statistics for all $124 + 153 = 277$ banks in the sample. The next two blocks of numbers report summary statistics for the 124 CN banks and the 153 USA banks. The first row in each block reports the number of efficient observations (i.e., the number of times the relevant performance measure equals 1). The next three rows in each block report the geometric averages,[11] standard deviations, and minima of the relevant estimates.

Several conclusions can be drawn from the results reported in Table 2. First, by construction, estimates of $\mathrm{CE}(x, y, w)$ obtained using the C-NC model can be no higher than those obtained using the NC-NC model. This is reflected in both the lower average CE score and the smaller number of efficient observations under the C-NC model. The estimates of $\mathrm{CE}(x, y, w)$ obtained using the C-NC model are on average $0.9282 - 0.6587 = 0.2695$ lower than those obtained using the NC-NC model; this translates into a percentage difference of $(0.9282 - 0.6587)/0.9282 = 29.03\%$.[12]

Second, also by construction, estimates of $\mathrm{RCE}^g(x, y, w)$ obtained using the C-NC model can be no higher than estimates obtained using the NC-NC model. This again shows up in both the lower average RCE score and the smaller number of efficient observations under the C-NC model. The estimates of $\mathrm{RCE}^g(x, y, w)$ obtained using the C-NC model are on average $0.9556 - 0.7297 = 0.2259$ lower than those obtained using the NC-NC model; this translates into a percentage difference of $(0.9556 - 0.7297)/0.9556 = 23.64\%$.[13]

[10] Briec and Kerstens (2009) discuss on the possibility of infeasibilities for general distance functions.

[11] The use of geometric averages guarantees that the multiplicative decomposition in (3) holds exactly.

[12] Taking the ratio of the CE estimates nets out the observed cost and reveals the difference in the estimated value of the MCF under convexity and nonconvexity.

Table 2
C-NC and NC-NC Estimates of $\mathrm{CE}(x, y, w)$, $\mathrm{RCE}^g(x, y, w)$ and $\mathrm{CMR}^g(y, w)$.

		C-NC				NC-NC			
		$\mathrm{CE}(\cdot)$	$\mathrm{RCE}^g(\cdot)$	$\mathrm{CMR}^g(\cdot)$	Infeas.	$\mathrm{CE}(\cdot)$	$\mathrm{RCE}^g(\cdot)$	$\mathrm{CMR}^g(\cdot)$	Infeas.
All 277 Banks	# Effic. Obs.	17	31	160		155	194	220	
	Geom. Mean	0.6587	0.7297	0.9027	8	0.9282	0.9556	0.9713	79
	Stand. Dev.	0.1539	0.1490	0.1264		0.1009	0.0847	0.0633	
	Min.	0.4054	0.4054	0.4517		0.5759	0.5759	0.6381	
	Li-test[a]	106.393	82.870	105.462					
	p-value	(0.000)	(0.000)	(0.000)					
124 CN Banks	# Effic. Obs.	8	19	22		66	91	87	
	Geom. Mean	0.6397	0.7934	0.8062	4	0.9280	0.9733	0.9535	79
	Stand. Dev.	0.1646	0.1258	0.1312		0.0925	0.0533	0.0789	
	Min.	0.4301	0.6034	0.4517		0.6381	0.7450	0.6381	
	KSW-test#1[a]		3.020						
	p-value		(0.003)						
	KSW-test#2[a]		0.852						
	p-value		(0.025)						
153 USA Banks	# Effic. Obs.	9	12	138		89	103	133	
	Geom. Mean	0.6745	0.6818	0.9893	4	0.9283	0.9415	0.9860	0
	Stand. Dev.	0.1437	0.1500	0.0394		0.1075	0.1021	0.0429	
	Min.	0.4054	0.4054	0.7495		0.5759	0.5759	0.6507	
	KSW-test#1[a]		236.459						
	p-value		(0.485)						
	KSW-test#2[a]		0.766						
	p-value		(0.225)						

[a] Exact p values are reported in round brackets underneath.

Third, estimates of $\mathrm{CMR}^g(y, w)$ obtained using the C-NC model can in theory be either higher or lower than those obtained using the NC-NC model. Table 2 reveals that in our application estimates of $\mathrm{CMR}^g(y, w)$ obtained using the C-NC model are on average $(0.9713 - 0.9027)/0.9713 = 7.06\%$ lower than estimates obtained using NC-NC model. Furthermore, the average value of the $\mathrm{CMR}^g(y, w)$ estimates obtained using the NC-NC (or C-NC) model for the 153 USA banks is close to one and also larger than the corresponding estimate for the CN banks. This indicates that the USA banks in our sample are cost-superior to the CN banks, and that some managers of CN banks may benefit by adopting the technology of the USA banks. We are only aware of a handful of other studies that use metafrontier methods to determine the inferiority or superiority of specific technologies. For instance, Sala-Garrido et al. (2011) evaluate four wastewater treatment technologies and find that one technology dominates all three others.

Fourth, there are 4 instances of infeasible solutions for each sub-sample of CN and USA banks when computing distances to selected technology-specific frontiers using the C-NC model. This represents about 3% of the sample. In contrast, for the NC-NC model, there are 79 instances of infeasible solutions among CN banks, while USA banks do not encounter any infeasible solutions at all. This represents about 28.5% of the sample.

To formally assess the differences in efficiency scores, we employ a nonparametric test initially proposed by Li (1996): refinements are due to Li et al. (2009), among others. This nonparametric test focuses on differences between entire distributions of efficiency scores instead of focusing on, for instance, differences in first moments (as, e.g., the Wilcoxon signed-rank test). It looks for differences between two kernel-based estimates of density functions f and g of a random variable x. The null hypothesis is that the two probability density functions (pdfs) are equal: $H_0 : f(x) = g(x)$ for all x. The alternative hypothesis is that they are not equal: $H_1 : f(x) \neq g(x)$ for some x.[14]

Results of these Li tests are reported in the first block of Table 2: we report the values of the Li test, and the exact p values are reported in round brackets underneath. We reject the null hypothesis that estimates of $CE(x, y, w)$ obtained using the C-NC model have the same distribution as estimates obtained using the NC-NC model: Li test statistic takes the value 106.393; and the exact p-value is 0.000. We also reject the null hypothesis that estimates of $RCE^g(x, y, w)$ obtained using the C-NC model have the same distribution as estimates obtained using the NC-NC model (p-value = 0.000). Finally, we reject the null hypothesis that estimates of $CMR^g(y, w)$ obtained using the C-NC model have the same distribution as estimates obtained using the NC-NC model (p-value = 0.000).

To formally test whether each technology-specific cost function for CN and USA banks separately is convex, we extend the convexity test proposed by Kneip et al. (2016) and further robustified by Simar and Wilson (2020a) for the production function to the cost function. Details about the initial and robustified version of the tests as well as some sensitivity analysis are found in Appendix B. The null hypothesis is that the cost function is convex in the outputs, while the alternative hypothesis is that the cost function is nonconvex in the outputs. The extension in Appendix B describes two tests, denoted as KSW-test#1 and KSW-test#2. KSW-test#1 involves computing the average of the test statistic across several sample splits. KSW-test#2 entails conducting a Kolmogorov–Smirnov test to assess the uniformity of the distribution of p-values across multiple sample splits.

The test results presented in Table 2 show that for the CN banks both KSW-test#1 and KSW-test#2 reject the convexity assumption and hence the cost function is nonconvex in outputs. By contrast, we cannot reject the convexity assumption for USA banks and hence the convexity assumption for the cost function seems suitable for USA banks. This should make applied researchers think harder about imposing convexity: to the best of our knowledge, the studies by Campos-Alba et al. (2020) and Pérez-López et al. (2016) are the only other studies using a NC-NC model in this cost metafrontier context (but these authors do

[13] Taking the ratio of the RCE estimates nets out the observed cost and reveals the difference in the estimated value of the TCF under convexity and nonconvexity.

[14] The test is valid for both dependent and independent variables. Observe that dependency is a characteristic of frontier estimators: i.e., cost efficiency levels depend on sample size, among others. We opt for the standard Li test over an adapted-Li test (Simar & Zelenyuk, 2006) since the statistical properties for meta-cost efficiency and the cost-oriented metatechnology ratio are not yet available. The *R* code for the Li-test can be found in the *np* package.

Table 3
C-NC and C-C Estimates of $\mathrm{CE}(x, y, w)$ and $\mathrm{CMR}^{g}(y, w)$.

		CE(·)			CMR^g(·)		
		C-NC	C-C	Difference	C-NC	C-C	Difference
All 277 Banks	# Effic. Obs.	17	10		160	21	
	Arith. Mean	0.6752	0.6099	0.0653	0.9127	0.8288	0.0838
	Stand. Dev.	0.1539	0.1310	0.0698	0.1264	0.1268	0.0809
	Min.	0.4054	0.3534	0.0000	0.4517	0.4474	0.0000
	Li-test[a]			2.926			65.863
	p-value			(0.004)			(0.000)
124 CN Banks	# Effic. Obs.	8	5		22	8	
	Arith. Mean	0.6582	0.6119	0.0462	0.8170	0.7601	0.0569
	Stand. Dev.	0.1646	0.1491	0.0507	0.1312	0.1170	0.0598
	Min.	0.4301	0.4227	0.0000	0.4517	0.4474	0.0000
153 USA Banks	# Effic. Obs.	9	5		138	13	
	Arith. Mean	0.6889	0.6082	0.0807	0.9901	0.8845	0.1057
	Stand. Dev.	0.1437	0.1148	0.0789	0.0394	0.1056	0.0890
	Min.	0.4054	0.3534	0.0000	0.7495	0.5302	0.0000

[a] Li test: exact p values are reported in round brackets underneath.

not make a comparison with the traditional C-NC model). Kerstens and Van de Woestyne (2021) do show that convexity is a disputable axiom within a traditional cost function context.

We believe ours is the first study to correctly compute the cost function associated with a nonconvex MPPS and compare the results obtained when the MPPS is formed as the union of either convex or nonconvex TPPSs. A first empirical conclusion is that constructing a nonconvex MPPS as the union of nonconvex TPPSs yields estimates of minimum cost that are about 29% higher than estimates obtained by constructing a nonconvex MPPS from convex TPPSs. These empirical results are in line with other studies that compare standard cost functions based on convex and nonconvex PPSs: an example includes Balaguer-Coll et al. (2007). Therefore, imposing convexity or not on TPPSs should be more seriously considered when computing cost functions associated with nonconvex MPPSs.

To assess the price of inappropriately convexifying the MPPS when computing the cost function, descriptive statistics for the associated potentially-affected estimates of $\mathrm{CE}(x, y, w)$ and $\mathrm{CMR}^{g}(y, w)$ are reported in the columns labeled C–C in Table 3: the acronym C–C indicates that the TPPSs and MPPS have all been convexified when computing the cost function. Note that the component $\mathrm{RCE}^{g}(x, y, w)$ is not affected by the convexification strategy: therefore, it is not reported in Table 3. Most of the column and row labels in Table 3 are self explanatory. The columns labeled "Difference" report the differences between the C-NC (unbiased) and the C–C (biased) results.

The arithmetic average difference between the C-NC and C–C estimates of $\mathrm{CE}(x, y, w)$ is only 0.6752 − 0.6099 = 0.0653 (less than 0.1), while the average difference between the C-NC and C–C estimates of $\mathrm{CMR}^{g}(y, w)$ is 0.9127 − 0.8288 = 0.0838 (slightly larger, but still less than 0.1).[15] Furthermore, this average difference between the C-NC and C–C estimates is lower for CN banks than for USA banks.

Li tests are once again applied to all 277 banks to test whether the two distributions of $\mathrm{CE}(x, y, w)$ estimates are equal and whether the two distributions of $\mathrm{CMR}^{g}(y, w)$ estimates are equal. We reject the null hypothesis that estimates of $\mathrm{CE}(x, y, w)$ obtained using the C-NC model have the same distribution as estimates obtained using the C–C model (test statistic = 2.926; p-value = 0.004). Moreover, we also reject the null hypothesis that estimates of $\mathrm{CMR}^{g}(y, w)$ obtained using the C-NC model have the same distribution as estimates obtained using the C–C model (test statistic = 65.863; p-value = 0.000). Thus, a second empirical conclusion is that both MCF and CMR seem to be affected by a convexification strategy. This suggests that the convexification strategy used by almost all authors when estimating cost metafrontiers should be abandoned.

In terms of policy relevance, the convexification strategy may cause policymakers to misinterpret the technological gaps among different group of producers. This could lead policymakers to make incorrect decisions in attempting to close these gaps. In terms of predictive accuracy, Jin et al. (2024) report in a context of anomaly detection slightly superior classification results for nonconvex relative to convex production models.

6. Conclusions

The seminal article of O'Donnell et al. (2008) considers a production possibilities metaset that is defined as the union of several underlying group-specific PPSs. The boundary of the metaset is referred to as a production metafrontier and the boundaries of the group-specific sets are referred to as group frontiers. O'Donnell et al. (2008) suggest estimating the metafrontier under the assumption that group-specific PPSs and the metaset are convex. Kerstens et al. (2019) develop some key results showing that both convex and nonconvex group-specific sets in general yield nonconvex metasets. This indicates that the convexification strategies that are commonly used when estimating production metafrontiers should not be pursued *a priori*, but ideally require empirical testing to confirm that they are innocuous.

This contribution has focused on the way convexification strategies are used in a cost function context: we have argued that production possibilities metasets are generally nonconvex, so cost metafrontiers are normally nonconvex in outputs. We have developed theoretical results for general PPSs and cost functions in this respect, and also for nonparametric specifications of the same sets and functions under a variety of assumptions. We have used the data from CN and USA banks in 2019 to explore the consequences of making incorrect assumptions about the convexity or nonconvexity of PPSs and associated cost functions. We focused on the consequences of a convexification strategy for estimates of cost efficiency (CE) and cost-oriented metatechnology ratios (CMRs). We found that estimates of CE and CMRs are sensitive to convexity assumptions. Since one counterexample is sufficient to invalidate an hypothesis, the reported results offer a clear case to reject the assumption that the convexification strategy suggested by O'Donnell et al. (2008) is empirically innocuous when estimating cost functions. Obviously, our results carry over immediately to the estimation of revenue functions. They also carry over to the estimation of short-run profit functions (see Briec et al. (2004) for relevant arguments).

It is now possible to see how our general results concerning sets and functions and on nonparametric estimators might be transposed

[15] Since here no decomposition must be preserved, we take arithmetic rather than geometric averages.

to alternative frontier methodologies. First, the transposition to alternative nonparametric frontier methods (e.g., conditional convex and nonconvex models, convex and nonconvex order-m models (see Daraio & Simar, 2007)) looks straightforward, but remains to be done. Second, it should be straightforward to assess the way convexification strategies are used in a deterministic parametric frontier context, but this also remains to be done. Third, a proper construction of a production possibilities metaset using stochastic frontier methods has recently been explored by Amsler et al. (2017): again, a transposition to a cost function context remains to be developed. Finally, constructing nonconvex production possibilities metasets using stochastic nonparametric methodologies seems possible (see, e.g., Afsharian (2017) who focuses on the so-called StoNED approach): again, the cost function case remains to be developed.

The findings of this study highlight critical implications for policymakers and managers in industries employing metafrontier studies. Convexification in cost metafrontiers can lead to biased efficiency estimates, which may misinform decision-making. Policymakers should reconsider regulatory frameworks that rely on such estimates to compare firms across different technological environments. Furthermore, technology-specific cost functions may well be nonconvex: this will ensure more accurate benchmarking. For managers, the results emphasize the importance of selecting appropriate frontier estimation methods. Firms should be cautious in adopting efficiency scores derived from convex models, since these may not reflect actual performance gaps. In industries where technological heterogeneity is prevalent, decision-makers should leverage nonconvex metafrontier models to identify performance gaps. This approach can help firms optimize resource allocation, improve competitiveness, and develop targeted investment strategies that align with their specific technological and market conditions.

To conclude, it is possible to outline further research that would be useful to evaluate how the many different metafrontier applications discussed in Section 1 are affected by the possibly incorrect assumption that the production possibilities metaset as well as the resulting cost function is convex. First, we have limited our empirical analysis of cost functions to the case where frontiers exhibit VRS: thus, our empirical analysis could be easily redone for the case where frontiers exhibit CRS. Second, it is useful to replicate our study with other data sets from banking and other sectors to see whether convexification creates a bias and to assess whether convexity of the group cost functions can be rejected. Third, it remains an open question whether metatechnology specific cost functions in the convex case can be computed via a single linear program. Fourth, developments in this contribution can obviously be generalized to the cases of the metatechnology-specific revenue function and to the case of the metatechnology-specific short-run profit function.

CRediT authorship contribution statement

Kristiaan Kerstens: Writing – review & editing, Writing – original draft, Visualization, Validation, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Christopher O'Donnell:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Ignace Van de Woestyne:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Shirong Zhao:** Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ejor.2025.05.048.

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